Fold a paper tray

FAMILY PROJECT 05 · OFTEN AGES 11–13 · READINESS FIRST

A flat plan. A solid idea.

Turn a paper net into an open tray model. Connect rectangular area to layers of volume, compare ratios, and express the same ideal volume as a fraction and decimal of a liter.

PILOT — not physically tested with families. Assembly, handling and learning outcomes are unverified. This paper model is not a usable liquid, food or load-bearing container.

Prepare with a parent

Use a sheet of plain paper at least 12 × 10 cm (Letter or A4 works), spare paper, centimeter ruler, pencil, four short pieces of tape, and adult scissors. A calculator is optional. Reuse supplies; no purchase is needed. The parent draws/checks the net and handles all cutting. Fold by hand; do not use a knife or scoring tool.

Work on a clear table under adult supervision. Keep scissors and tape scraps away from younger children; stop if materials are mouthed. Do not fill the tray with water, food, hot materials or heavy objects. It is a geometry model only.

Readiness: measure a 6 cm line and explain 6 × 4 as four rows of six. Model this with a drawing if needed. Begin with the net and base area; ratios and liters can wait. Parents lead; no speed requirement or child login.

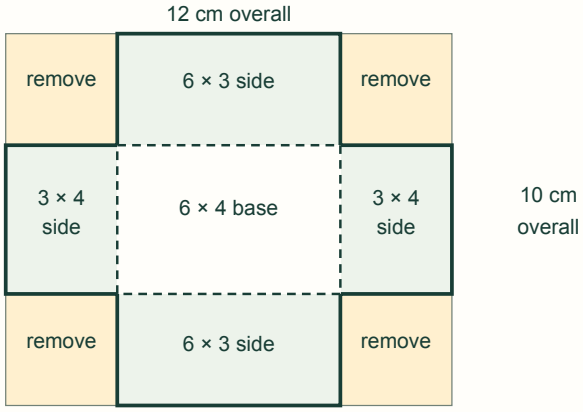
Preparation list

Draw one net, then cut as directed on page 2

Part	Planned dimensions
Starting rectangle	12 × 10 cm
Center base	6 × 4 cm
Two long side panels	6 × 3 cm each
Two short side panels	4 × 3 cm each
Four corner squares to remove	3 × 3 cm each

Draw, cut and fold the net

1. Draw a 12 cm-wide by 10 cm-tall rectangle. From each edge, measure inward 3 cm and draw a parallel line. The center is 6 × 4 cm because 12 − 3 − 3 = 6 and 10 − 3 − 3 = 4.
2. The adult cuts out the large rectangle, then removes the four 3 × 3 cm corner squares. **Keep the four side panels attached to the base.** No glue tabs are needed; tape will join the upright corners.
3. Fold along the four base edges (dashed lines below). Raise the sides approximately perpendicular to the base. An adult can hold adjacent sides while you place tape across the outside of each corner. Use short pieces that do not pull the shape out of square.
4. Check the base and side heights with a ruler. If the model is distorted, flatten and re-fold it or redraw. Compare measured dimensions with the plan; do not report a planned size as a measured result.



All diagram dimensions are centimeters. Schematic only: draw and measure your own net, not the displayed/printed picture. Dashed lines = folds; solid cross outline = cuts.

> Ask: “Which flat rectangle becomes the base? Which measurement becomes the height? Where would a top panel be?” This model has no top.

Area, volume and a constant ratio

Count squares, then imagine layers

Sketch a 6-by-4 grid on spare paper. It contains 24 squares of area 1 cm². A 1 cm-high layer above that base has ideal volume 24 cm³; three layers give **6 × 4 × 3 = 72 cm³**. A square centimeter measures area; a cubic centimeter measures volume. No small cubes or liquid are needed.

We ignore paper thickness, tape, rounded folds and distortion in this ideal rectangular-prism calculation. It is **not a measured capacity** of your folded tray. Actual inside dimensions may differ.

Using 1 liter = 1,000 cm³, 72 cm³ = 72/1,000 L = 9/125 L = **0.072 L**. Also, 1 cm³ = 1 mL, so the ideal volume is 72 mL. Do not fill the paper model to check this conversion.

Change the base while keeping its shape

The base length-to-width ratio is 6:4 = 3:2. One ratio unit corresponds to 2 cm in the original base. Use a table instead of beginning with cross-multiplication.

Length and width change by the same factor; height stays 3 cm

Base length	Base width	Ratio units	Ideal volume
3 cm	2 cm	1 cm per unit	18 cm³
6 cm	4 cm	2 cm per unit	72 cm³
9 cm	6 cm	3 cm per unit	162 cm³

The 9-by-6 base multiplies each original base dimension by 1.5, so base area and volume (with unchanged height) multiply by 1.5² = 2.25. **This is not a uniformly enlarged 3D model**: its height did not change.

How much paper stays in the model?

Base plus sides: 24 + 2 × 18 + 2 × 12 = **84 cm²**. Or start with 12 × 10 = 120 cm² and remove four 9 cm² corners: 120 − 36 = 84 cm². Tape is excluded. Ideal volume 72 cm³ and net area 84 cm² answer different questions.

My tray planning sheet

Use drawings, talk or writing. A parent may read and record. Keep the model and worked page available, and note any supports used.

1. Observe: planned base = 6 × 4 cm; planned side height = 3 cm. My measured base = ____ × ____ cm; side height = ____ cm. What changed during folding?

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2. Explain: sketch one 1 cm-high layer. Why is 24 cm² an area but 72 cm³ a volume?

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3. Ratio: keep a 3:2 base ratio. If length is 12 cm, width is ____ cm. With height still 3 cm, ideal volume is ____ cm³ = ____ L. Show the common factor.

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4. Paper: show two ways to find the area of the original open tray's net. Why should you not add a lid area?

Optional decimal budget: imagine paper costs $0.30 per whole sheet and tape for one model costs $0.04. Budget one sheet per model; ignore leftover paper. How much for one model? Three models? These are invented prices, not a shopping quote.

Use planned sizes for questions 2–4, not your measured sizes. No mastery score, name or upload. Private observation sheet can record what was modeled and what the learner explained independently.

Parent answers and evidence limits

Answers: 1. Measurements vary; record real observations, not invented trial results. 2. 6 × 4 = 24 unit squares cover the base; three 1 cm-high layers have volume 24 × 3 = 72 cm³. 3. 12 ÷ 3 = 4 cm per ratio unit; width = 2 × 4 = 8 cm. Volume = 12 × 8 × 3 = 288 cm³ = 288/1,000 = 0.288 L. The base dimensions are twice the original, so volume is four times 72 when height stays fixed. 4. 24 + 36 + 24 = 84 cm², or 120 − 4 × 9 = 84 cm². The tray is open: no lid belongs in this net.

Budget: $0.30 + $0.04 = $0.34 per model; 3 × $0.34 = $1.02. Equivalently, 3 × 30 + 3 × 4 = 102 cents. Actual supplies may be reused; no cost or purchase is required.

Access: offer larger drawings, one direction at a time, adult cutting/scribing and a calculator when reasoning about shape is the goal. **Instruction:** trace the base, sketch its unit squares, then connect three equal layers to multiplication. For ratios, return to 3 cm and 2 cm, then double both. Retain models as needed; support is not evidence of unaided calculation.

Dyscalculia, math anxiety, attention/working-memory difficulties and reading/language difficulties are distinct and can overlap. Slow folding or unit confusion does not diagnose any of them. This is not treatment; discuss persistent learning concerns with a teacher or qualified professional.

Selected-source teaching note

Siegler, R., Carpenter, T., Fennell, F., Geary, D., Lewis, J., Okamoto, Y., Thompson, L., & Wray, J. (September 2010). *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade* (NCEE 2010-4039). Institute of Education Sciences. Official guide · Full PDF.

Recommendation 4 (pp. 35–41) advises developing the meaning of ratio/rate/proportion strategies before cross-multiplication; pp. 39–40 illustrate ratio tables. It has a **minimal evidence** rating: limited qualifying research does not establish that a specific table or home project will improve learning. The guide spans K–8; this is not a single age-11–13 sample or high-school evidence. Our table connects paired lengths by the same multiplier, an editorial adaptation of the teaching approach.

This original pack and selected-source note are not a systematic review or experiment. The cited ratio recommendation does not validate our volume sequence, paper construction, diagnosis-specific effectiveness or home learning gains. Geometry, dimensions and example prices are our own content; physical testing remains pending.

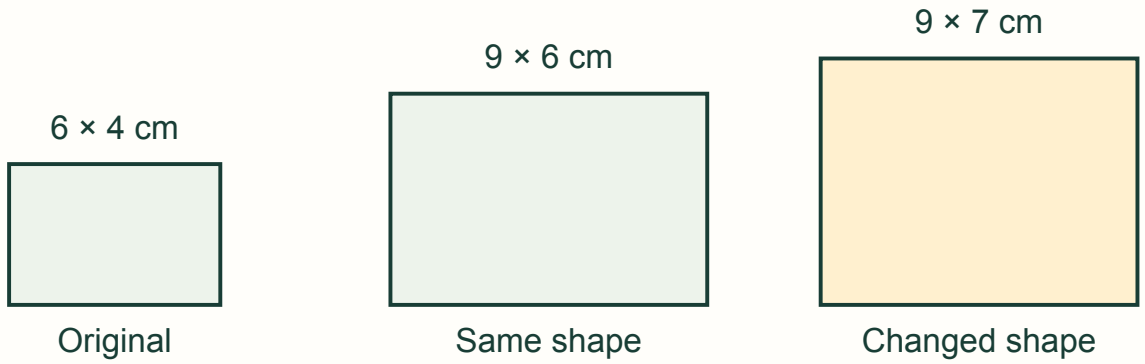
More parent guidance · Choose another starting point

Optional extension: keep the shape, not the difference

Use the same spare paper, pencil and ruler; no more cutting or purchases. Parent leads. Start when the learner can follow the 3:2 table on page 3. These are flat base drawings, not extra tray-building instructions.

Model one correct enlargement first

Draw a 6 × 4 cm rectangle. To make its length 9 cm while keeping the shape, find the common multiplier: **9 ÷ 6 = 1.5**. Multiply the width too: **4 × 1.5 = 6 cm**. Draw 9 × 6 cm. Check: 6 ÷ 4 = 9 ÷ 6 = 1.5; in both rectangles the length is one and a half times the width.



Schematic, not a full-size measuring template

Then compare a clearly labeled non-example

Incorrect for preserving this shape: adding 3 cm to both sides gives 9 × 7 cm. The original difference is 6 − 4 = 2 cm; the new difference is 9 − 7 = 2 cm. The difference stayed equal, but the ratio did not: 9/7 is not 3/2. Adding the same length is not the same as multiplying by the same factor.

> Ask: “What did we keep the same: the difference or the ratio? Show where each multiplier appears.” If needed, return to doubling: 6 × 4 becomes 12 × 8, not 12 × 10.

Addition can still be a valid ratio strategy

Build up **matching 3:2 pairs**: (3, 2), (6, 4), (9, 6). Add 3 cm to the length *and 2 cm to the width* each time. This keeps the ratio, unlike adding 3 cm to both. Compare the table with multiplying both original dimensions by 1.5. No cross-multiplication is needed here.

Try, explain, then revisit

Use the model or table as needed. Parent may read, scribe or offer a calculator. This is practice, not a timed test or a diagnostic screen.

1. Finish the pair: for a base ratio of 3:2, a 15 cm length needs a ____ cm width. Show a ratio table or explain the multiplier.

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2. Check a claim: “8 × 6 cm has the same shape as 6 × 4 cm because both side differences are 2 cm.” Agree or disagree? Give a ratio-based reason and repair the width for an 8 cm length.

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3. Later, change the numbers: keep the same 3:2 ratio, but make the width 7 cm. What length is needed? Explain without assuming the multiplier is a whole number.

Parent answers and next step

1. 10 cm: 15 ÷ 3 = 5, then 2 × 5 = 10. Or build matching (3, 2) pairs five times. **2.** Disagree: 8/6 = 4/3, not 3/2. For an 8 cm length, width = 8 × 2/3 = 16/3 = 5⅓ cm. Fraction placement can be approximate on paper; keep the written answer exact. **3.** 10.5 cm: 7 ÷ 2 = 3.5, then 3 × 3.5 = 10.5. Check 10.5 ÷ 7 = 1.5. Relative to the original 6 × 4 base, both dimensions multiply by 1.75.

Record access separately from teaching: “parent read the prompt” differs from “parent supplied the ×1.5 multiplier.” Both are allowed. If the learner keeps a difference instead of a ratio, model one matching-pair table, then invite a fresh example. If fractional arithmetic obscures the idea, use 12 × 8 first. Keep supports available; do not infer mastery or retention from these few examples.

Selected-source scope for this extension

Siegler et al. (September 2010), *Developing Effective Fractions Instruction for Kindergarten Through 8th Grade*, IES, NCEE 2010-4039; full citation and links on page 5. Recommendation 4, pp. 35–41: develop ratio meanings before cross-multiplication; pp. 38–40 show buildup strategies, ratio tables, multiplicative relationships and scaling. Its rating is **minimal evidence**. Page 36 describes small strategy case studies that did not test instructional improvement, alongside limited intervention evidence. This K–8 synthesis is not a single ages-11–13 study or evidence for teens or dyscalculia treatment.

Our rectangle comparison, prompts and follow-up are original editorial adaptations, not a tested intervention, systematic review or experiment. The source does not establish effectiveness of these home materials. The physical pack remains an untested PILOT.